Representation Theory of Symmetric Groups and Related Algebras (11 - 20 Dec 2017)

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Venue:







3 Prince George's Park Singapore 118402 ims.nus.edu.sg

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Dear participants, welcome to our institute here in Singapore!

Registration

Upon arrival at IMS, participants who receive funding support from us are requested to check in with our program administrators Ms Jia Ling LEE or Ms Eileen TAN at the IMS Admin Office (building next to the IMS Auditorium) as we have some registration paperwork needed for you to complete.

General Information

In this programme booklet, you will find the names of the speakers and the time the presentations will take place. Presentations include time for Q&A sessions.

We encourage speakers not to use their own laptop computers since changing the audio visual setups in the auditorium may cause slight technical delays at the beginning of the session. Presenters could bring their PowerPoint or Adobe presentations on a flash drive and upload their files into the desktop computer (Windows platform) in the auditorium before the start of their session.

Lunch is offered at IMS on the days when the schedule indicates "Lunch Reception at IMS". Other meals are at self expenses and self arrangements.

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Food and drinks are prohibited in the auditorium because the flooring is carpeted.

Thank you.



Representation Theory of Symmetric Groups and Related Algebras

(11 – 20 Dec 2017)

Venue:

Institute for Mathematical Sciences, Auditorium 3 Prince George's Park Singapore 118402

Thursday, 14 Dec 2017

Time	Title	Speaker	
09:10am - 09:30am	Registration		
09:30am - 10:30am	Are there symmetric group crystals?	Arun Ram University of Melbourne, Australia	
10:30am - 11:30am	On bases of Specht modules corresponding to 2- column partitions	Sinead Lyle University of East Anglia, UK	
11:30am - 01:30pm	Lunch Break		
01:30pm - 02:30pm	Integral Schur-Weyl duality for partition algebras	Stuart Martin University of Cambridge, UK	
02:30pm - 03:30pm	Schur-Weyl duality and dominant dimension	Ming Fang Chinese Academy of Sciences, China	
03:30pm - 04:00pm	Coffee Break		
04:00pm - 05:00pm	The super-strong linkage principle for symmetric groups	Christopher Bowman University of Kent, UK	
05:45pm - 06:30pm	Transport from IMS to National Library, Bugis		
06:30pm - 07:30pm	Public Lecture: The Liar Game: Truths & Proofs from Euclid to Turing	Mark Wildon Royal Holloway, University of London, UK	
Friday, 15 Dec 2017			
09:20am - 09:30am	Registration		
09:30am - 10:30am	The first cohomology of Specht modules and extensions of symmetric powers [Part I]	Stephen Donkin University of York, UK	
10:30am - 11:30am	The first cohomology of Specht modules and extensions of symmetric powers [Part II]	Haralampos Geranios University of York, UK	
11:30am - 01:30pm	Lunch Break		

Time Title Speaker Jun Hu On the centers of cyclotomic quiver Hecke 01:30pm - 02:30pm Beijing Institute of Technology, China algebras Crystals, higher-level Fock spaces, and **Emily Norton** 02:30pm - 03:30pm representations of Cherednik algebras Max Planck Institute for Mathematics, Germany Coffee Break 03:30pm - 04:00pm Hyohe Miyachi On the Mackey formula for Hecke algebras and 04:00pm - 05:00pm GGOR category O Osaka City University, Japan Saturday, 16 Dec 2017 Registration 09:20am - 09:30am Lifting stable to derived equivalences with Changchang Xi 09:30am - 10:30am applications to Broué's abelian defect group Capital Normal University, China conjecture Stacey Law 10:30am - 11:30am p'-branching for symmetric groups University of Cambridge, UK Lunch Reception at IMS 11:30am - 12:30pm Jonathan Brundan 12:30pm - 01:30pm Representations of the oriented Skein category University of Oregon, USA Monoidal categories associated with strata of flag **Euiyong Park** 01:30pm - 02:30pm manifolds University of Seoul, Korea 02:30pm - 03:00pm Coffee Break Ana Paula Santana The Kostant form of U(sln+) and the Borel-Schur 03:00pm - 04:00pm algebra University of Coimbra, Portugal Monday, 18 Dec 2017 09:20am - 09:30am Registration Amit Hazi 09:30am - 10:20am A linkage principle for Soergel bimodules University of Leeds, UK Mark Wildon 10:20am - 10:45am The mod 2 homology of the simplex Royal Holloway, University of London, UK William O'Donovan 10:45am - 11:10am Young modules and Nakayama's conjecture Royal Holloway, University of London, UK Group Photo and Lunch Reception at IMS 11:10am - 01:30pm Liron Speyer

Representation Theory of Symmetric Groups and Related Algebras

Decomposable Specht modules

01:30pm - 02:30pm

The University of Virginia, USA

Representation Theory of Symmetric Groups and Related Algebras

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Time	Title	Speaker	
02:30pm - 03:30pm	Some new decomposition numbers for the Iwahori-Hecke algebra of type B	Louise Sutton National University of Singapore, Singapore	
03:30pm - 04:00pm	Coffee Break		
04:00pm - 05:00pm	Content systems and deformations of KLR algebras	Andrew Mathas University of Sydney, Australia	
Tuesday, 19 Dec 2017			
09:20am - 09:30am	Registration		
09:30am - 10:30am	Computing symmetric group decomposition numbers in the anti-spherical module	Geordie Williamson University of Sydney, Australia	
10:30am - 11:30am	Irreducible characters and Sylow p-subgroups	Eugenio Giannelli University of Cambridge, UK	
11:30am - 01:30pm	Lunch Break		
01:30pm - 02:30pm	Diagrammatic categories of type Q	Jonathan Kujawa University of Oklahoma, USA	
02:30pm - 03:30pm	Invariants of Kazhdan-Lusztig cells	Edmund Howse National University of Singapore, Singapore	
03:30pm - 04:00pm	Coffee Break		
04:00pm - 05:00pm	A character-theoretic proof of a result on defect zero blocks for symmetric and alternating groups	David Hemmer University at Buffalo, SUNY, USA	
Wednesday, 20 Dec 2017			
09:20am - 09:30am	Registration		
09:30am - 10:30am	RoCK blocks of symmetric groups and generalized Schur algebras	Alexander Kleshchev University of Oregon, USA	
10:30am - 11:30am	Schurifying quasihereditary algebras	Robert Muth Tarleton State University, USA	
11:30am - 01:30pm	Lunch Break		
01:30pm - 02:30pm	On block algebras of Hecke algebras of classical type	Susumu Ariki Osaka University, Japan	
02:30pm - 03:30pm	Perfect isometries and basic sets	Jean-Baptiste Gramain University of Aberdeen, UK	
03:30pm - 04:00pm	Coffee Break		
04:00pm - 05:00pm	Weight two blocks of double covers of symmetric groups	Matthew Fayers Queen Mary University of London, UK	

On block algebras of Hecke algebras of classical type

Susumu Ariki

Osaka University, Japan

ABSTRACT

We consider block algebras of finite dimensional Hecke algebras of classical type, where we consider the unequal parameter case as well as the equal parameter case in type B. Using recent advances in the theory of cyclotomic quiver Hecke algebras, such as the Brundan-Kleshchev isomorphism theorem [2], the Chuang-Rouquier sl_2 -categorification theorem [3], the Kang-Kashiwara cyclotomic categorification theorem [5], combined with techniques from representation theory of finite dimensional algebras, such as Rickard's theorems on derived equivalence and stable equivalence [7], Krause's theorem on representation type [6], we have determined the representation type of each of the block algebras when the base field is algebraically closed of an odd characteristic. As an application, we show that if a block algebra (over an algebraically closed field of odd characteristic) is of finite representation type then it is a Brauer tree algebra whose Brauer tree is a straight line without exceptional vertex. This verifies an old observation by Uno, Geck and myself on Hecke algebras in classical type. Here, the cellularity plays an important role and we apply Ohmatsu's theorem and the Rickard star theorem.

References

- Ariki S. Representation type for block algebras of Hecke algebras of classical type, Adv. Math. 317 (2017), pp. 823–845.
- Brundan J. and Kleshchev A. Blocks of cyclotomic Hecke algebras and Khovanov-Lauda algebras, Invent. Math. 178 (2009), pp. 451–484.
- [3] Chuang J. and Rouquier R. Derived equivalences for symmetric groups and sl₂categorification, Annals of Math. 167 (2008), pp. 245–298.
- [4] Erdmann K. and Nakano D. Representation type of Hecke algebras of type A, Trans. Amer. Math. Soc. 354 (2002), pp. 275–285.
- [5] Kang S.-J. and Kashiwara M. Categorification of highest weight modules via Khovanov-Lauda Rouquier algebras, Invent. Math. 190 (2012), pp. 699–742.
- [6] Krause H. Stable equivalence preserves representation type, Comment. Math. Helv. 72 (1997), pp. 266–284.
- [7] Rickard J. Derived categories and stable equivalence, J. Pure Appl. Algebra 61 (1989), pp. 303–317.

The super-strong linkage principle for symmetric groups

Chris Bowman

University of Kent, UK

ABSTRACT

We will discuss a "super strong linkage principle" which provides degree-wise upper bounds for graded decomposition numbers of symmetric groups (and more general complex reflection groups).

We will go on to review a new approach to understanding the modular representations of Hecke algebras of complex reflection groups. This approach allows us to construct lots of different graded cellular bases of these Hecke algebras. These new cellular bases allow us to restrict our attention from the whole module category to a subcategory of representations which is both rich in structure, but also understandable over fields of sufficiently large characteristic. We hence provide higher-level generalisations of the familiar "generic behaviour" and Kazhdan–Lusztig theory and settle Martin–Woodcock's conjecture.

References

- [1] C. Bowman, The many graded cellular bases of Hecke algebras, arXiv:1702.06579.
- [2] C. Bowman, A. Cox, and L. Speyer, A family of graded decomposition numbers for diagrammatic Cherednik algebras, Int. Math. Res. Not. IMRN 9 (2017), 2686–2734.
- [3] C. Bowman and A. G. Cox, Modular decomposition numbers of cyclotomic Hecke and diagrammatic Cherednik algebras, arXiv:1706.07128.
- [4] C. Bowman and L. Speyer, Kleshchev's decomposition numbers for diagrammatic Cherednik algebras, Trans. Amer. Math. Soc. (to appear).
- [5] B. Webster, Rouquier's conjecture and diagrammatic algebra, arXiv:1306.0074

Representations of the Oriented Skein Category

Jonathan Brundan

University of Oregon, USA

ABSTRACT

The oriented skein category OS(z,t) is a ribbon category which underpins the definition of the HOMFLY-PT invariant of an oriented link, in the same way that the Temperley-Lieb category underpins the Jones polynomial. In this talk I will explain how the representation theory of OS(z,t) can be developed using a highest weight approach, starting from the well-known representation theory of the Hecke algebra.

The first cohomology of Specht modules and extensions of symmetric powers [Parts I and II]

STEPHEN DONKIN AND HARALAMPOS GERANIOS

University of York, UK

ABSTRACT

Working over a field of odd characteristic p we describe explicitly the dimension of the first cohomology $H^1(\Sigma_r, \operatorname{Sp}(\lambda))$, of a Specht module $\operatorname{Sp}(\lambda)$ for the symmetric group Σ_r , for a partition λ of r. The problem has been considered earlier by several people, including Hemmer, Kleshchev, Nakano and Weber.

We proceed in several steps. We choose $n \ge r$ and write B for the Borel subgroup of lower triangular matrices in the general linear group $\operatorname{GL}_n(K)$, over an algebraically closed field K of characteristic p. Then $H^1(\Sigma_r, \operatorname{Sp}(\lambda))$ has an interpretation as the space $\operatorname{Ext}_B^1(S^r E, K_\lambda)$ of B-module extensions between the rth symmetric power $S^r E$, of the natural module E for $\operatorname{GL}_n(K)$, and the one dimensional B-module K_λ , defined by λ . We then pass to the hyperalgebra of B and realise such an extension in terms of the actions of divided powers operators corresponding to negative roots for $\operatorname{GL}_n(K)$. The explicit conditions required lead to the notions of: extension sequence, coherent triple of extension sequences and extension multi-sequence. We then proceed by describing all extension sequences, then all ways of knitting together extension sequences to form coherent triples and finally all ways in which these may be put together to form extension multi-sequences. The first of these procedures is straightforward but the last two are not. As well as a complete solution of the initial problem there are some simple general results which come out of our analysis.

Throughout the analysis the notion of a James partition plays a central role. This is the notion used by James to describe Specht modules with non-zero invariant (and in fact our approach via divided powers, gives a short proof of James's Theorem).

The first part of the presentation, given by Stephen Donkin, will focus on the transition from the original problem to the divided powers operators and the concrete conditions that define an extension multi-sequence sequence.

The second part of the presentation, given by Haralampos Geranios, will be concerned with the manipulation of extension multi-sequences and our explicit solution of the original problem as well as consequences of a general nature.

Schur-Weyl duality and dominant dimension

Ming Fang

Chinese Academy of Sciences, China

ABSTRACT

Schur-Weyl duality that relates the representation theories of general linear and symmetric groups, has been extensively studied and extended in various contexts. Dominant dimension, a homological dimension that was introduced by Nakayama in his complete homology theory, has attained much less attention and remains mysterious on the other hand. In this talk, I will explain a crucial role of dominant dimension in the study of Schur-Weyl duality and demonstrate some applications of this approach to Schur-Weyl duality and to the theory of dominant dimension itself.

Weight two blocks of double covers of symmetric groups

MATT FAYERS

Queen Mary University of London, UK

ABSTRACT

The decomposition number problem for the symmetric groups remains apparently intractable for the time being. Nevertheless, we have some nice results in special cases. On of these is Richards's combinatorial formula for the decomposition numbers for blocks of defect 2.

Our knowledge of the decomposition numbers for the double covers of the symmetric groups is relatively poor, despite that fact that many of the tools we use for symmetric groups (such as the Brundan–Kleshchev modular branching rules) have been shown to have analogues for double covers. I will talk about my attempts to find an analogue of Richards's formula for defect 2 blocks of double covers.

Irreducible characters and Sylow *p*-subgroups

Eugenio Giannelli

University of Cambridge, UK

ABSTRACT

The relevance of the McKay conjecture in the representation theory of finite groups has led to study how irreducible characters of a finite group G restrict to Sylow *p*-subgroups. In this talk I will present some recent work on this topic. I will particularly highlight the combinatorics involved in the case where G is the symmetric group.

Perfect Isometries and Basic Sets

JEAN-BAPTISTE GRAMAIN^a

^a University of Aberdeen, UK

ABSTRACT

Basic sets can be a powerful tool in modular representation theory of finite groups, sometimes helping in constructing Brauer characters or in reducing the problem of determining decomposition matrices.

In this talk, I will present some fairly old results on basic sets for the symmetric and alternating groups ([1], [2]), and show how the methods and results can be generalised to the double Schur covers of these groups when the characteristic is odd ([3]). In all these groups, we exhibit basic sets by using perfect isometries between blocks of complex characters.

This is joint work with Olivier Brunat (Université Paris 7).

References

- Brunat, O. and Gramain, J.-B., A basic set for the alternating group, J. Reine Angew. Math. 641 (2010), pp. 177–202.
- Brunat, O. and Gramain, J.-B., A 2-basic set of the alternating group, Arch. Math. 94(4) (2010), pp. 301–309.
- [3] Brunat, O. and Gramain, J.-B., Basic sets for the double covering groups of the symmetric and alternating group in odd characteristic, Preprint.

A linkage principle for Soergel bimodules

Amit Hazi

University of Leeds, UK

ABSTRACT

The diagrammatic category of Soergel bimodules is a linear, additive, monoidal category with deep connections to Kazhdan–Lusztig theory and representation theory. One of the central problems in this area is to explicitly characterize the indecomposable Soergel bimodules. This problem is closely related to calculating decomposition numbers for reductive groups and symmetric groups. I will describe a linkage principle for Soergel bimodules in characteristic p over an affine Weyl group, which in some sense decomposes them into Soergel bimodules for the p-affine Weyl subgroup. This provides new lower bounds on just how small the indecomposables can be.

A character-theoretic proof of a result on defect zero blocks for symmetric and alternating groups

DAVID HEMMER

University at Buffalo, SUNY, USA

ABSTRACT

While trying to solve the problem of which columns of the character table of symmetric groups have no zeroes, we ran into an interesting property about characters vanishing on elements of order p.

We managed to prove this result for symmetric and alternating groups and set to work on proving it in general. Unfortunately we later discovered the result was indeed known for finite groups in general, but every known proof requires modular representation theory. So we are left with a new character theoretic proof of a stronger result for symmetric and alternating groups and a cautionary tale about premature excitation! This talk will present the saga in all its gory detail.

Invariants of Kazhdan–Lusztig cells

Edmund Howse

National University of Singapore

ABSTRACT

Lusztig has described the partitioning of a Coxeter group W into left, right and two-sided cells with respect to a weight function. This description relies on certain equivalence relations that are calculated in the corresponding Iwahori– Hecke algebra \mathcal{H} , and the resulting cells afford representations of both W and \mathcal{H} .

One of the main problems in the theory of Kazhdan–Lusztig cells is their classification for all finite Coxeter groups. In this talk, I will discuss general techniques that have led to the classification of cells of the Coxeter group of type B_n with respect to a certain choice of weight function.

On the centers of cyclotomic quiver Hecke algebras of type A

Jun Hu

Beijing Institute of Technology, Beijing, 100081, P.R. China

ABSTRACT

For any symmetric generalized Cartan matrix C, any β in the positive root lattice with height n and any integral dominant weight Λ , one can associate a quiver Hecke algebra $\mathscr{R}_{\beta}(K)$ and its cyclotomic quotient $\mathscr{R}^{\Lambda}_{\beta}(K)$ over K. It has been conjectured that the natural map from $\mathscr{R}_{\beta}(K)$ to $\mathscr{R}^{\Lambda}_{\beta}(K)$ maps the center of $\mathscr{R}_{\beta}(K)$ surjectively onto the center of $\mathscr{R}^{\Lambda}_{\beta}(K)$. A similar conjecture claims that the center of the affine Hecke algebra of type A maps surjectively onto the center of the cyclotomic Hecke algebra of type $G(\ell, 1, n)$ over K. In this talk we will show that the second conjecture is equivalent to the first conjecture in type A.

RoCK blocks of symmetric groups and generalized Schur algebras

ALEXANDER KLESHCHEV

University of Oregon, USA

ABSTRACT

We review the proof of Turner's conjecture which gives a local description of RoCK blocks of symmetric groups in terms of certain generalized Schur algebras. This result is joint with Anton Evseev.

Diagrammatic categories of type Q

JONATHAN KUJAWA

University of Oklahoma, USA

ABSTRACT

We will discuss several combinatorial categories where the morphisms are given by diagrammatics. They allow us to concretely describe several categories which arise naturally in representation theory. In particular, this includes the representations of Lie superalgebras of type Q and the representations of the Sergeev algebras (aka the spin representations of the symmetric groups). This work is joint with J. Comes and G. Brown.

p'-branching for symmetric groups

STACEY LAW

University of Cambridge, UK

ABSTRACT

Let p be any prime and n be any natural number. Let χ be an ordinary irreducible character of the symmetric group \mathfrak{S}_n whose degree is coprime to p. We bound the number of p'-irreducible constituents of the restriction of χ to \mathfrak{S}_{n-1} . This generalizes work of Ayyer, Prasad and Spallone [1] for the prime p = 2. This is joint work with E. Giannelli and S. Martin.

References

 A. Ayyer, A. Prasad, and S. Spallone, Odd partitions in Young's lattice. Sémin. Lothar. Comb. 75 (2016), Article B75g.

On bases of Specht modules corresponding to 2-column partitions

SINÉAD LYLE

University of East Anglia, UK

ABSTRACT

In this talk, we consider Specht modules and simple modules corresponding to 2-column partitions. Decomposition numbers for Specht modules corresponding to such partitions have long been known; as have homomorphims between them. For each such partition λ , we look at certain sets of standard λ -tableaux which can be defined combinatorially in terms of paths and which naturally label a basis of the simple module D^{λ} . We also prove that the *q*-character of D^{λ} can be described in terms of this sets. We consider the extension to 3-column partitions.

This is joint work with Melanie de Boeck, Anton Evseev and Liron Speyer.

Integral Schur–Weyl duality for partition algebras

Chris Bowman^a, Steve Doty^b, and <u>Stuart Martin^c</u>

^aUniversity of Kent, UK ^bLoyola University, Chicago, USA ^cUniversity of Cambridge, UK

ABSTRACT

Partition algebras were discovered (independently) in the 1990s by Paul P. Martin and Vaughan Jones, as generic centralisers of the natural action of the Weyl group W of GL(V) on the tensor algebra of V, where V is a finitedimensional vector space. By construction, partition algebras satisfy a Schur-Weyl duality with the group algebra of W, at least over a field of characteristic zero, in which case the group algebra is semisimple. In this joint work with Bowman and Doty I explore why Schur-Weyl duality still holds, even when the underlying field is replaced by an arbitrary ring (of any characteristic). In particular, it holds over the integers. This extends a result of Gibson (1980) on generalised doubly-stochastic matrices.

References

- R.A. Brualdi, Permanent of the product of doubly stochastic matrices, Proc. Cambridge Philos. Soc. 62 (1966), 643–648.
- [2] P.M. Gibson, Generalized doubly stochastic and permutation matrices over a ring, Linear Algebra Appl. 30 (1980), 101–107.
- [3] T. Halverson and A. Ram, Partition algebras, European J. Combin. 26 (2005), 869–921.
- [4] E.C. Johnsen, Essentially doubly stochastic matrices. I. Elements of the theory over arbitrary fields, Linear Algebra and Appl. 4 (1971), 255–282.
- [5] H.-C. Lai, On the linear algebra of generalized doubly stochastic matrices and their equivalence relations and permutation basis, Japan J. Appl. Math. 3 (1986), 357–379.

Content systems and deformations of KLR algebras

ANDREW MATHAS

University of Sydney, Australia

ABSTRACT

Since they were first defined in 2008, the KLR algebras and their cyclotomic quotients have generated a vast literature. In type A, the cyclotomic KLR algebras are relatively well understood, partly because Brundan and Kleshchev's graded isomorphism theorem allows us to tap into the (ungraded) representation theory of the cyclotomic Hecke algebras of type A. Obtaining bases for the other types of cyclotomic KLR algebras has so far proved to be difficult. I will describe a new approach, via a graded deformation of the cyclotomic KLR algebras of types $A_{e+1}^{(1)}$ and $C_e^{(1)}$, that makes it much easier to calculate in these algebras. As a consequence we give a uniform description of graded cellular bases in both cases.

This is joint work with Anton Evseev.

On the Mackey formula for Hecke algebras and GGOR category O

HYOHE MIYACHI

Osaka City University, Japan

ABSTRACT

In the representation theory of finite groups, the Mackey formula plays very important roles such as Green's vertex theory, etc. The finite complex reflection group W has natural flat deformations, called their associated Hecke algebras H(W). We report yet another Mackey formula in this set up as well as the associated category O=O(W) defined by Ginzburg-Guay-Opdam-Rouquier.

This is a joint work with T. Kuwabara and K. Wada.

Schurifying quasihereditary algebras

Alexander Kleshchev^a and <u>Robert Muth</u>^b

^aUniversity of Oregon, USA ^bTarleton State University, USA

ABSTRACT

For any based quasihereditary superalgebra A over an integral domain of characteristic zero, we construct an associated generalized Schur algebra $S^A(n,d)$. Some important algebras arise in this way; in proving a conjecture of Turner's [3], Evseev and Kleshchev have recently shown that RoCK blocks of Hecke algebras are Morita equivalent to generalized Schur zigzag algebras [1]. We show (under some constraints) that $S^A(n,d)$ is based quasihereditary, generalizing Green's codeterminant basis for the classical Schur algebra [2]. Finally, we describe decomposition numbers of standard modules for $S^A(n,d)$ in terms of those of A and the classical Schur algebra. This is joint work with Alexander Kleshchev.

References

- A. Evseev, A. Kleshchev, Blocks of symmetric groups, semicuspidal KLR algebras and zigzag Schur-Weyl duality, arXiv:1603.03843.
- [2] J. A. Green, Combinatorics and the Schur algebra, J. Pure Appl. Algebra 88 (1993), 89–106.
- [3] W. Turner, Rock blocks, Mem. Amer. Math. Soc., 202 (2009), no. 947.

Crystals, higher-level Fock spaces, and representations of Cherednik algebras

Emily Norton

Max Planck Institute for Mathematics, Germany

ABSTRACT

Information about representations of cyclotomic rational Cherednik algebras, such as finite-dimensionality, is encoded in two commuting crystals on a higher level Fock space. These two crystals arise from induction and restriction functors to different types of parabolics. I will focus on the crystal which comes from a categorical action of a Heisenberg algebra on the Category O of the Cherednik algebra. The categorical action is due to Shan-Vasserot, and the crystal is due to Losev, who also introduced a way to compute it using wall-crossings. However, no one had found a direct combinatorial rule for the action of the crystal operators on an arbitrary charged multipartition. I will explain the rule, which builds on previous work of Thomas Gerber, as well as some representation-theoretic consequences. This is joint work with Thomas Gerber.

Young Modules and Nakayama's Conjecture

William O'Donovan

Royal Holloway, University of London, UK

ABSTRACT

The indecomposable summands of a Young permutation module over a field of prime characteristic are given by the Young modules. In this talk, we provide a brief account of their main representation-theoretic properties. We then explain how the machinery of Young modules may be utilised to give a proof (using only the representation theory of the symmetric group and elementary block theory) of Nakyama's Conjecture on the blocks of the symmetric group. If time permits, we outline how these methods can be adapted to prove a theorem of Hemmer on the blocks of the signed Young Modules.

Monoidal categories associated with strata of flag manifolds

Masaki Kashiwara^a, Myungho Kim^b, Se-jin Oh^c, and $\underline{Euiyong Park}^{d}$

^aResearch Institute for Mathematical Sciences, Kyoto University, Japan ^bKyung Hee University, Korea ^cEwha Womans University, Korea ^dUniversity of Seoul, Korea

ABSTRACT

We construct a monoidal category $\mathscr{C}_{w,v}$ which categorifies the doubly-invariant algebra $^{N'(w)}\mathbb{C}[N]^{N(v)}$ associated with Weyl group elements w and v. It gives, after a localization, the coordinate algebra $\mathbb{C}[\mathcal{R}_{w,v}]$ of the open Richardson variety associated with w and v. The category $\mathscr{C}_{w,v}$ is realized as a subcategory of the graded module category of a quiver Hecke algebra R. When $v = \mathrm{id}$, $\mathscr{C}_{w,v}$ is the same as the monoidal category which provides a monoidal categorification of the quantum unipotent coordinate algebra $A_q[\mathfrak{n}(w)]_{\mathbb{Z}[q,q^{-1}]}$ given by Kang-Kashiwara-Kim-Oh. We show that the category $\mathscr{C}_{w,v}$ contains special determinantial modules $\mathsf{M}(w_{\leq k}\Lambda, v_{\leq k}\Lambda)$ for $k = 1, \ldots, \ell(w)$, which commute with each other. When the quiver Hecke algebra R is symmetric, we find a formula of the degree of R-matrices between the determinantial modules $\mathsf{M}(w_{\leq k}\Lambda, v_{\leq k}\Lambda)$. When it is of finite ADE type, we further prove that there is an equivalence of categories between $\mathscr{C}_{w,v}$ and \mathscr{C}_u for $w, u, v \in \mathsf{W}$ with w = vuand $\ell(w) = \ell(v) + \ell(u)$. This talk is based on arXiv:1708.04428.

Are there Symmetric group crystals?

Arun Ram

University of Melbourne, Australia

ABSTRACT

Several recent conversations about the Kronecker problem (positive combinatorial expressions for the decomposition of tensor products of irreducible symmetric group representation in characteristic 0) made me feel that it would be useful to think about the possibility of a theory of crystals for symmetric group representations (a graph based categorification of the character ring of the symmetric group). Initial explorations have been fascinating. I will give a summary of what I have learned about what such a theory would need to look like.

The Kostant form of $\mathfrak{U}(sl_n^+)$ and the Borel-Schur algebra

Ana Paula Santana and Ivan Yudin

CMUC, University of Coimbra, Portugal

ABSTRACT

In 1981, S. Donkin proved that the Schur algebra for the general linear group has finite global dimension. This led to the problem of describing explicit projective resolutions of the Weyl modules for the Schur algebra.

Let R be a commutative ring and denote by $\mathfrak{U}_n^+(R)$ the Kostant form over R of the universal enveloping algebra of the Lie algebra of $n \times n$ complex nilpotent upper triangular matrices. In this talk I will explain the construction of functors that map (minimal) projective resolutions of the rank-one trivial $\mathfrak{U}_n^+(R)$ -module to (minimal) projective resolutions of rank-one modules for the Borel-Schur algebra. Using Woodcock's Theorem, from these resolutions one can easily obtain projective resolutions of the Weyl modules for the Schur algebra.

Decomposable Specht modules

LIRON SPEYER

The University of Virginia, USA

ABSTRACT

I will give a brief survey of the study of decomposable Specht modules for the symmetric group and its Hecke algebra, which includes results of Murphy, Dodge and Fayers, and myself. I will then report on an ongoing project with Louise Sutton, in which we are studying decomposable Specht modules for the Hecke algebra of type B indexed by 'bihooks'.

Some new decomposition numbers for the Iwahori-Hecke algebra of type B

LOUISE SUTTON

National University of Singapore

ABSTRACT

The Decomposition Number Problem for the cyclotomic KLR algebra in higher levels is far from being completely understood. Restricting to level two, a particularly nice family of Specht modules for the Iwahori-Hecke algebra of type B are those labelled by hook bipartitions, denoted $S_{((n-m),(1^m))}$. We will first give an overview of the decomposition numbers corresponding to Specht modules labelled by hook bipartitions in quantum characteristic at least three, where $S_{((n-m),(1^m))}$ has at most 4 composition factors. In quantum characteristic two, the decomposition submatrix associated with hook bipartitions is more complex; $S_{((n-m),(1^m))}$ has at most 2m + 1 composition factors. We will present ongoing work to determine these decomposition numbers.

The mod 2 homology of the simplex

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ABSTRACT

For each natural number t there is a raising map sending a d-set to the formal sum of all (d - t)-sets that it contains. These maps define a chain complex on the two-row Young permutation modules in characteristic 2. When t = 1 this chain complex is exact; equivalently, the d-simplex has no non-zero homology. I will describe some of the 'higher' homology groups. In particular, we get an explicit construction of certain two-row Young modules and a new construction of the basic spin representations of the symmetric groups.

Computing symmetric group decomposition numbers in the anti-spherical module

GEORDIE WILLIAMSON

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ABSTRACT

As everyone at this conference knows, it is not easy to compute decomposition numbers for symmetric groups. I will try to explain why results I obtained a few years ago ("torsion explosion") suggest that this problem is perhaps harder than we thought. The moral seems to be that questions of an arithmetic nature (e.g. is this Fibonacci number prime?) are encoded in this problem. For these reasons I believe the answer lies beyond combinatorics.

On the other hand, one can hope that there is a beautiful combinatorial theory for large p (suitably interpreted). A new method of calculating tilting characters for reductive groups (work of Achar, Elias, Libedinsky, Losev, Makisumi, Riche and myself) allows one to calculate many never before seen decomposition numbers. The algorithm is rather complicated (and the proof that it works runs to hundreds of pages), but I will try to give an outline. Staring at the output led Lusztig and I to a new conjecture for certain three row decomposition numbers, which resembles a discrete dynamical system: https://www.youtube.com/watch?v=Ru0Zys1Vvq4.

Lifting stable to derived equivalences with applications to Broué's abelian defect group Conjecture

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ABSTRACT

Motivated by two major conjectures in representation theory: Broué's Abelian Defect Group Conjecture and Auslander-Alperin Conjecture on stable equivalences, we consider the question of how to lift a stable equivalence of Morita type to a derived equivalence. In this talk, we show that, for Frobenius-finite algebras, each individual stable equivalence of Morita type does induce a special derived equivalence introduced in [2], where an algebra is called Frobenius-finite if its Frobenius part is representation-finite. During the talk, we also show how our result can be applied to give partial answers to the two conjectures.

The contents of the talk are taken from a paper [1] jointly with W. Hu.

References

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